Planar panels and planar supporting beams in architectural structures

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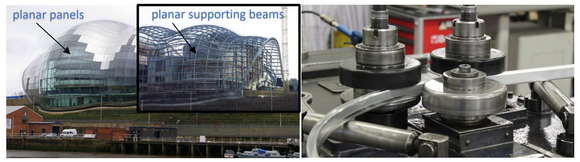
Paper

Introduction

We investigate geometric properties and modeling space of quad meshes with planar faces (panels) and planar parameter lines (supporting beams). This planarity is a major benefit in architectural design and building construction. We discuss the design of planar quad ("PQ") meshes with one family of planar polylines ("P"), or with both families of planar polylines ("PP"). We work in the space of planes and with appropriate transformations of that space. We also incorporate further properties of near-rectangular faces ("Conical meshes").

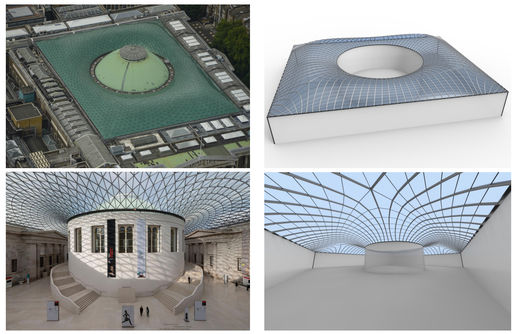


Overview: Combination of mesh constraints

1. P+PQ	2. PP+PQ	3. P+Conical	4. PP+Conical
Shape approximation	Form-finding		Explicit computation
Optimization	Weighted face image	Laguerre transformation	Explicit weighted face image
	Modeling in the space of **PLANES**		

1. P+PQ meshes

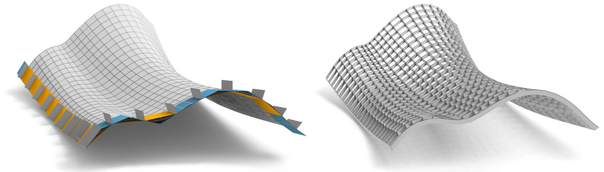
P+PQ meshes have a comparatively mild restriction and can be designed with standard global optimization approaches [Tang et al. 2014]. They are also capable of (at least locally) approximating given shapes.



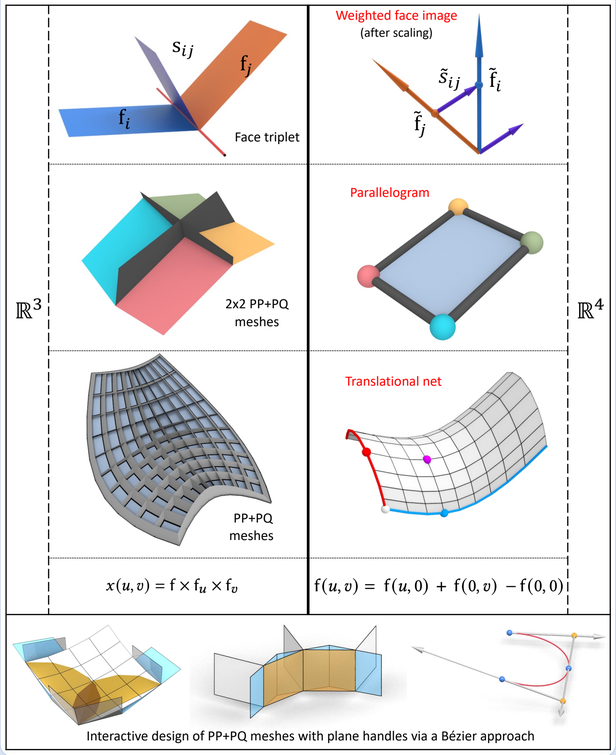
Architectural rationalization of the roof on the Great Court at the British Museum

2. PP+PQ meshes

Our method designs PP+PQ meshes from the space of planes. The colored plane handles can interactively control the mesh and its support structures. This can be viewed as a Bézier approach when projected into $\mathbb{R}^4$.



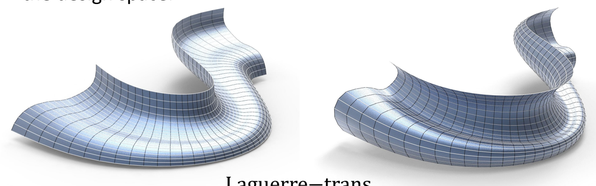
The plane coordinates in $\mathbb{R}^4$ formulate a translational mesh as a weighted face image. The translational property in this dual space can simplify the design of PP+PQ meshes. We use the plane handles as a control structure to interactively design the two boundary Bézier curves of the translational mesh.



Interactive design of PP+PQ meshes with plane handles via a Bézier approach

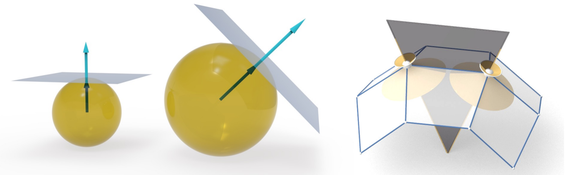
3. P+Conical meshes

The geometric property of P plus Conical is invariant under Laguerre transformations (L-trans). L-trans can re-permutate the planes containing the planar parameter lines to enlarge the design space.

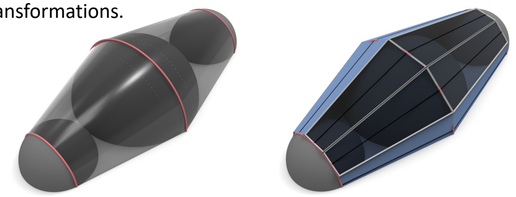


$$\begin{bmatrix} planes \xrightarrow{\text{Laguerre-trans}} planes \\ normals \xrightarrow{\text{Möbius-trans}} normals \end{bmatrix}$$

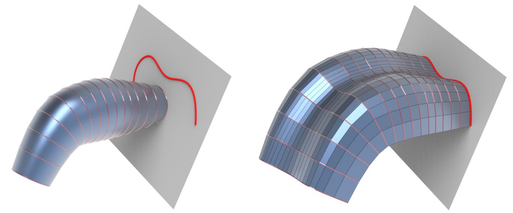
L-trans can map spheres to spheres, planes to planes, and keep their oriented contacts. Hence, they preserve conical meshes. L-trans can also preserve planar principle curves.



We use semi-discrete Channel surfaces for initial access to P+Conical meshes. The semi-discrete Channel surfaces are suitable for both Laguerre transformations and parallel transformations.

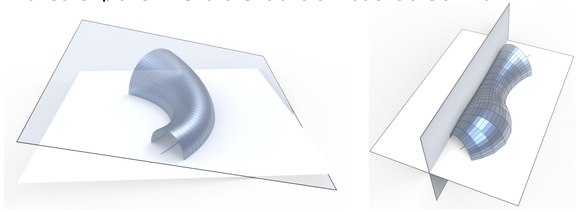


Parallel transformations can be used to enlarge the design space by editing a planar profile and propagating.

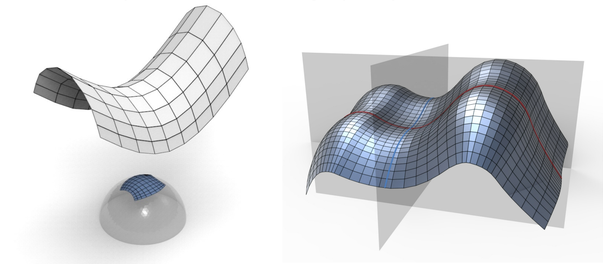


4. PP+Conical meshes

PP+Conical meshes can be taken as a special case of P+Conical meshes. The initial shape can be accessed via a Channel surface with spheres centered on a plane and tangential to another plane. The further transformations are similar.

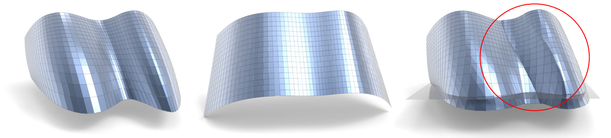


We can fully explore the design space of PP+Conical meshes by explicitly calculating the weighted face image via its unit counterpart on the Gauss sphere. The interactive design only needs to input two curves on major principal sections.



Discussion

We work in the space of planes with the insights of projective geometry because simple global optimization approaches will fail; e.g. optimizing PP+PQ meshes towards PP meshes with nearly rectangular faces.



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